STAT 5 Final Study Guide

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Purpose of this Guide

This document outlines the most important topics and formulas to review for the STAT 5 final exam. Use it as a roadmap to organize your studying and focus on essential concepts.

Study Topics and Formulas

Descriptive Statistics

- **Sample Mean:** $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- **Sample Variance:** $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$
- **Standard Deviation:** $s = \sqrt{s^2}$

*Know how to compute and **interpret** these summary statistics.*

Probability

- $P(A^c) = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cap B) = P(A|B) \cdot P(B)$

Be comfortable applying these rules in different scenarios, including when events are independent or disjoint

Normal Distribution

- **Standard Normal:** $Z = \frac{X - \mu}{\sigma}$, where X follows a $N(\mu, \sigma^2)$
- **Empirical Rule:** 68%, 95%, 99.7% chance within 1σ, 2σ, 3σ of μ.

Be able to standardize data and use the empirical rule.

Sampling and Study Design

- Distinguish between sampling methods: simple random, stratified, cluster, convenience.
- Understand controlled experiments vs. observational studies.

Recognize implications for statistical inference.

Point Estimates

- **One Mean:** $\hat{\mu} = \overline{x}$
- **One Proportion:** $\hat{p} = \frac{X}{n}$, X being the number of subjects in the samples that have a specific characteristic.
- **Two Proportions:** $\hat{p}_1 - \hat{p}_2 = \frac{X_1}{n_1} - \frac{X_2}{n_2}$, X_1 and X_2 being the number of subjects in the sample 1 and 2 that have a specific characteristic.

Understand what point estimators represent and how to compute them.

Confidence Intervals

- **One Mean:** $\overline{x} \pm z^* \frac{\sigma}{\sqrt{n}}, \quad \overline{x} \pm t^* \frac{s}{\sqrt{n}}$
- **One Proportion:** $\hat{p} \pm z^* SE(\hat{p})$
- **Two Proportions:** $(\hat{p}_1 - \hat{p}_2) \pm z^* SE(\hat{p}_1 - \hat{p}_2)$
- $SE(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- $SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

Be able to interpret confidence intervals and margin of error.

Hypothesis Tests

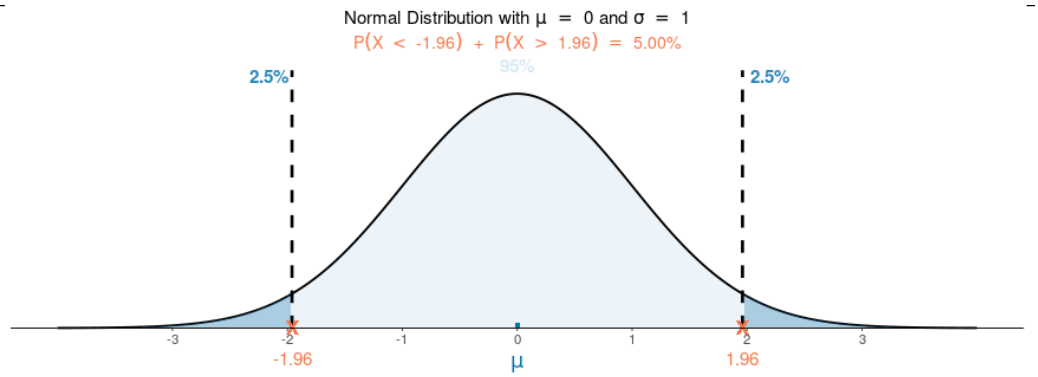
- **One Mean:** $Z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}, \quad t = \frac{\overline{x}-\mu_0}{s/\sqrt{n}}$
- **One Proportion:** $Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
- **Two Proportions:** $Z = \frac{(\hat{p}_1-\hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$, where $\hat{p} = \frac{x_1+x_2}{n_1+n_2}$

Know how to set up hypotheses, calculate test statistics, and interpret p-values.

Significance Level and Statistical Errors

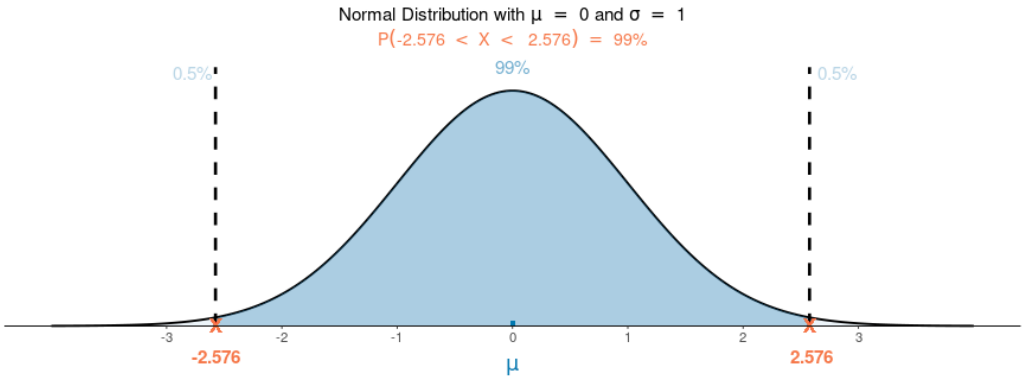
- Understand the meanings of significance level (α), Type I error, and Type II error.

Be able to define and recognize the consequences of errors in hypothesis testing.



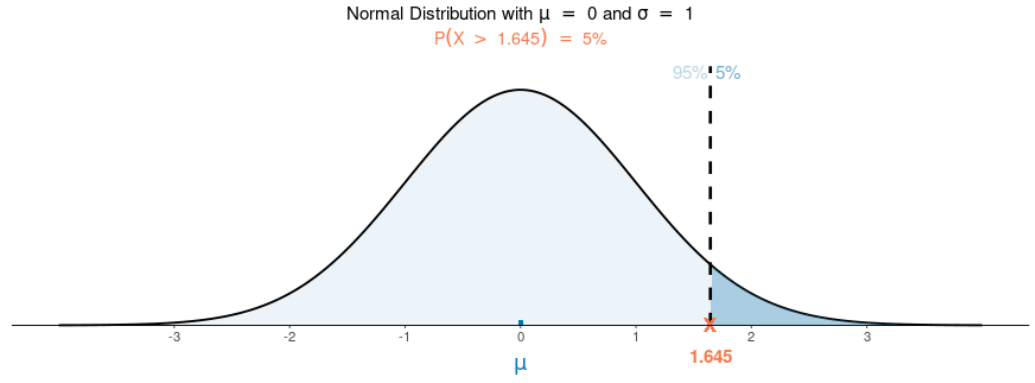
Normal Probability (Tail Probability):

Mean μ	Std. Dev. σ	Value of x1	Value of x2	Probability P(X < -1.96) + P(X > 1.96)	Lower z-Score	Upper z-Score
0	1	-1.96	1.96	5.00%	-1.96	1.96



Normal Percentile (Two-Tailed):

Mean μ	Std. Dev. σ	Central Percent	Value of x1	Value of x2
0	1	99%	-2.576	2.576



Normal Percentile (Upper Tail):

Mean μ	Std. Dev. σ	Percent in Upper Tail	Value of x
0	1	5%	1.645

Figure 1: Plots for p-value extraction

Regression

- $\hat{y} = b_0 + b_1 x$
- $b_1 = r\frac{s_y}{s_x}$, $b_0 = \overline{y} - b_1\overline{x}$
- $R^2 = \frac{\text{Var}(y) - \text{Var}(e)}{\text{Var}(y)} = r^2$ for simple linear regression.

Interpret the regression line, slope, intercept, and R^2.

Reading and Interpreting Data Tables

- Extracting relevant proportions or probabilities from contingency tables.
- Translating tabular information into meaningful statistical statements.

Assumptions for Statistical Inference

- Know when inferential methods are valid: random sampling, independence, sample size/normality.

Communicating Statistical Findings

- Explaining results in plain language, especially for non-technical audiences.
- Connecting statistical output to real-world questions.

Advice for Effective Studying

- Focus on understanding the "why" behind each concept, not just memorizing formulas.
- Practice stating hypotheses and interpreting statistical results in context.
- Be able to explain statistical reasoning in clear, everyday language.
- Review worked examples and try to solve a variety of practice problems.
- Don't hesitate to ask questions if any concept is unclear!

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