STAT 131 | **Final Exam** | **2024 Spring**

Thursday 12:00-13:30, June 13, 2024

This exam contains 6 pages (including this cover page) and 3 questions. The total number of possible points is 35 + 10 extra credit points.

- This is a closed-book exam. You may use a sheet of prepared notes, if you wish, but you may not share materials.
- If a question seems ambiguous or confusing, please ask me to clarify it. Unless a problem states otherwise, you must **show your work**. There are blank worksheets at the end of the test if you need more room for this.
- It is to your advantage to write your solutions as clearly as possible, and to $\boxed{\text{box}}$ answers I might not find.
- For full credit, answers must be given in **closed form** with no unevaluated sums, integrals, maxima, unreduced fractions. Wherever possible, **simplify**.

Do not write in the table below. Good luck.☺

Student Name (print): ______________________________

Question	Points	Score
1	10	____
2	10	____
3	15	____
Total:	35	

1. (10 points) **Transformation**

   Suppose a continuous random variable Y has the following pdf:

$$f(y) = Ce^{-y^2}1(y > 0), \text{ for } y \in \mathbb{R}. \tag{1}$$

   (a) (5 pts) Determine the constant C in the pdf of Equation (1).

   (b) (5 pts) Let $X = (2Z - 1)Y$ where $Z \sim Ber(0.5)$ and $Z \perp Y$. Find the CDF of X.

   (c) (5 extra credit pts) Suppose $Y_1, Y_2, ..., Y_n \overset{iid}{\sim} f$. Define $Y_{(n)} = \max\{Y_1, ..., Y_n\}$. Find the pdf of $Y_{(n)}$. (If you cannot find C in (a), you can write your answers with the undetermined C.)

Points earned: ______ **out of 10 points**

2. (10 points) **Conditional Expectation**

   Suppose $X \sim N(\mu, 1)$, and $Y|X = x \sim N(x, \sigma^2)$, where μ and σ^2 are two constants.

   (a) (5 pts) Find $E[XY]$.

   (b) (5 pts) Find $f_{X|Y}(x|y)$ and $E[X|Y]$.

   (c) (5 extra credit pts) Find $E[e^X|Y]$.

Points earned: ______ **out of 10 points**

3. (15 points) **Convergence of Random Variables**

   Suppose $Z_1, Z_2, ..., Z_n \overset{iid}{\sim} Unif(-1, 1)$.

   (a) (5 pts) Find the mean and the variance of the sample mean $\frac{1}{n}\sum_{i=1}^{n} Z_i$.

   (b) (5 pts) Find the limit of $\frac{1}{n}\sum_{i=1}^{n} Z_i^2$ as $n \to \infty$. (To earn full credits, you should specify the mode of convergence.)

   (c) (5 pts) Suppose $X \sim Bin(100, \pi)$, where π is a constant. Apply the central limit theorem to approximate the probability

   $$P(48 \leq X \leq 52).$$

   (Hint: your answer should be a function of π.)

 Points earned: ______ out of 15 points

Blank worksheet

Points earned: ______ out of 0 points

Common distributions

Name	Notation	pdf/pmf	Range	Mean	Variance
Beta	$Beta(a, b)$	$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$	$x \in (0, 1)$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$
Bernoulli	$Ber(p)$	$f(x) = p^x(1-p)^{1-x}$	$x \in \{0, 1\}$	p	$p(1-p)$
Binomial	$Bin(n, p)$	$f(x) = \binom{n}{x} p^x(1-p)^{n-x}$	$x \in \{0, 1, .., n\}$	np	$np(1-p)$
Exponential	$Exp(\lambda)$	$f(x) = \lambda e^{-\lambda x}$	$x \in \mathbb{R}_+$	$1/\lambda$	$1/\lambda^2$
Gamma	$Ga(a, b)$	$f(x) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}$	$x \in \mathbb{R}_+$	a/b	a/b^2
Normal	$N(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$x \in \mathbb{R}$	μ	σ^2
Poisson	$Pois(\lambda)$	$f(x) = \frac{\lambda^x}{x!} e^{-\lambda}$	$x \in \{0, 1, ...\}$	λ	λ
Uniform	$Unif(a, b)$	$f(x) = \frac{1}{b-a}$	$x \in (a, b)$	$(a+b)/2$	$(b-a)^2/12$

Points earned: ______ out of 0 points