Mathematics Review for STAT 131

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Introduction

This review covers essential mathematical concepts necessary for success in STAT 131. Mastery of these topics will help you understand probability theory and statistical methods more effectively.

Topics covered:

- Functions, limits, continuity
- Differentiation (e.g., product rule, quotient rule, chain rule)
- Integration (by parts and by substitution)
- Functions of two variables and double integrals
- Infinite series, summations, maxima, and minima

It is recommended that you complete this review before the second class to assess your mathematical background.

1. Problems

1. Completing the Square

Express the quadratic expression $4x^2 + 8x + 7$ in the form:

$$a(x + b)^2 + c$$

where a, b, and c are constants.

Hint: Factor out the leading coefficient and complete the square.

2. Logarithmic Identities

Express the following in terms of $\log(x)$ and $\log(y)$:

(a) $\log(x^2)$

(b) $\log(x/y)$

Key Formula:

Logarithm Rules

$$\log(a^b) = b\log(a), \quad \log\left(\frac{a}{b}\right) = \log(a) - \log(b).$$

3. Geometric Series

Evaluate the following sums:

(a) $\sum_{i=0}^{n} \frac{1}{3^i}$

(b) $\sum_{i=0}^{\infty} \frac{1}{3^i}$

Key Formula:

Geometric Series Formula

$$\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}, \quad \text{for } |r| < 1.$$

4. Integration by Parts

Evaluate:

$$\int_0^1 xe^x \, dx.$$

Integration by Parts Formula:

Integration by Parts

$$\int u \, dv = uv - \int v \, du.$$

5. Integration by Substitution

Evaluate:

$$\int_0^1 xe^{x^2} \, dx.$$

Substitution: Let $u = x^2$, then $du = 2x \, dx$.

6. Double Integrals

Evaluate:

$$\int\int_{x^2+y^2 \leq 1} dxdy.$$

Tip: Convert to polar coordinates where $x^2 + y^2 = r^2$.

7. Differentiation

Differentiate the following functions with respect to x:

(a) e^{-x^2}

(b) $\log(x^5)$

(c) $x^2 e^{-x}$

(d) $\int_0^{x^2} ye^y \, dy$

Key Rules:

Differentiation Rules

$$\frac{d}{dx}e^x = e^x, \quad \frac{d}{dx}\log(x) = \frac{1}{x}, \quad \frac{d}{dx}\int_a^x f(t) \, dt = f(x).$$

8. Sketching Graphs

Sketch rough plots of the following functions:

(a) $f(x) = e^{-x}$, for $x \in \mathbb{R}$.

(b) $f(x) = x^2$, for $x \in \mathbb{R}$.

(c) $f(x) = e^{-x^2}$, for $x \in \mathbb{R}$.

Tip: Identify key features such as intercepts, asymptotes, and concavity.

9. Matrix Operations

Consider the matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

(a) Compute the determinant $\det(A)$.

(b) Find the inverse A^{-1}.

Key Formula:

Determinant and Inverse

$$\det(A) = ad - bc, \quad A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$