# Midterm Review STAT 131

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## Introduction

- **Midterm Date and Time:**
  The STAT 131 Midterm is scheduled for **Friday, January 31st, 2025**.
- **Scope of the Midterm:**
  The exam will cover material from **Chapters 1 through 3/4** as outlined in the course syllabus.
- **Purpose of this Document:**
  This review provides a partial set of practice problems designed to help you prepare for the midterm. It focuses on material covered during **Weeks 1 through 4**.
- **Important Note:**
  This is *not* a comprehensive review. Additional preparation from the textbook, lecture notes, and problem sets is essential.

**Tips**

- Review the problem-solving techniques discussed in class, especially for counting, conditional probability, and Bayes' rule.
- Pay close attention to formulas and definitions. Make sure you understand how to apply them to solve problems.
- Practice showing all steps clearly, as partial credit will be awarded for correct reasoning, even if the final answer is incorrect.
- Check Summary 1 at **https://antonio-aguirre.com/teaching/**

# I. Probability via Counting

'Los Pericos' offers 10 types of tacos, each with unique ingredients. Three friends order tacos completely randomly and independently.

**Problem Statement:** What is the probability that at least two of the friends order the same type of taco?

**Tips**

- How does the total sample space looks like?
- How does the outcomes of interest look like?

## II. Conditional Probability and Independence

Five friends gather to celebrate Antonio's birthday. Naturally, there's a piñata involved, duh. Each friend takes one swing at the piñata, one after another. Each swing can result in either a "Hit" (**H**), meaning the piñata takes a blow, or a "Miss" (**T**), meaning they just flail aimlessly at the air. Each swing is independent from each other and all are equally likely to hit or miss.

Antonio records the outcomes in sequence. For instance, the outcome "**HHTTT**" means:

- The first friend landed a hit.
- The second friend also landed a hit.
- The last three friends swung valiantly but missed.

**Problem Statement:**

1. What is the probability that at least one swing was a Hit?
2. Suppose Antonio tells you that at least two of the five swings were Hits. What is the probability that all five swings were Hits?
3. Now, Antonio reveals that the first two swings were definitely Hits. What is the probability that all five swings were Hits?

**Tips**

- Recall that
$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}, \quad \text{provided } \Pr(B) > 0$$
- For events $A$ and $B$:
$$\Pr(A \cap B) = \Pr(A|B) \cdot \Pr(B)$$
- And, events $A$ and $B$ are *independent* if:
$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

## III. Bayes' Rule

In a large international gathering celebrating Norteño music—an unexpected fusion of Mexican and German music—the following is known:

- 80% of all Mexicans in attendance are extroverted.
- 15% of all Germans in attendance are extroverted.
- 10% of the attendees are Germans, while the remaining 90% are Mexicans.

**Problem Statement:** At the gathering, you meet someone who is not extroverted. What is the probability that this person is German?

**Tips**

- Start by defining the events of interest.
- Write down all the probabilities provided in the problem:
- Apply Bayes' Rule.

## IV. Bayes' Rule and Independence

In a vibrant Mexican festival featuring music and dance, there are two types of performers: talented musicians and amateurs. Consider the following events:

- $G$: The event that a performer is a talented musician.
- $A$: The event that the performer makes a mistake during their performance this week.
- $B$: The event that the performer makes a mistake during their performance next week.

Assume the following probabilities are known:

$$P(G) = g, \quad P(A \mid G) = P(B \mid G) = p_1, \quad P(A \mid G^c) = P(B \mid G^c) = p_2 \quad (p_1 < p_2)$$

Additionally, conditional on $G$, the events $A$ and $B$ are independent.

**Problem Statement:**

1. Explain intuitively whether $A$ and $B$ are independent.
2. Find $P(G|A^c)$ and $P(B|A^c)$.

## V. Discrete Distributions

Consider a lottery ticket where each ticket has a probability p of being a winning ticket, independently of other tickets. A gambler purchases 3 tickets.

**Problem Statement:**

1. Determine the distribution of the number of winning tickets among the 3 tickets purchased.
2. Prove that the probability of having at least one winning ticket is:

$$3p - 3p^2 + p^3$$

in two distinct ways:

(a) Using the inclusion-exclusion principle (you will need to define events).

(b) Using the complement rule and the PMF of a suitable known distribution discussed in class.

**Tips**

- For **1.**, recognize the distribution! (What is random in this problem? )
- For **Part 2. (a)**, recall the inclusion-exclusion formula for probabilities of overlapping events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

# VI. Probability Mass Function (PMF)

Consider a random variable X with the following probability mass function (PMF):

$$P(X = x) = c \cdot x, \quad \text{for } x = 1, 2, \ldots, 5, \quad \text{and } P(X = x) = 0 \text{ otherwise.}$$

**Problem Statement:** Determine the value of the constant c such that $P(X = x)$ is a valid probability mass function.

**Tips**

- Recall that for a PMF, the sum of all probabilities must equal 1:

$$\sum_x P(X = x) = 1$$

## VII. Discrete Distribution

A fair coin is tossed 10 times independently.

**Problem Statement:** Determine the probability mass function (PMF) of the number of heads obtained in the 10 tosses.

Tips

- What is random in this problem?
- Is this a recognizable distribution?