STAT 131 Formula Sheet

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Definitions

- **Probability Function:** A function P that assigns a numeric value between 0 and 1 to each event in a sample space.
- **Event:** A subset of the sample space Ω.
- **Disjoint Events:** Two events A and B are disjoint if $A \cap B = \emptyset$.
- **Independence:** Two events A and B are independent if:

$$P(A \cap B) = P(A)P(B).$$

Axioms of Probability Functions

- $0 \leq P(A) \leq 1$ for any event A.
- $P(\Omega) = 1$, where Ω is the sample space.
- If $A_1, A_2, \ldots$ are disjoint events, then:

$$P\left(\bigcup_{j=1}^{\infty} A_j\right) = \sum_{j=1}^{\infty} P(A_j).$$

Properties of Probability Functions (Consequences of Axioms)

- $P(A^c) = 1 - P(A)$ (**Complement Rule**).
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (**Addition Rule**).
- If $A \subseteq B$, then $P(A) \leq P(B)$ (**Monotonicity**).
- **Multiplication Rule:**

$$P(A \cap B) = P(A \mid B)P(B).$$

- **Bayes' Rule:**
$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}.$$

- **Law of Total Probability:**
$$P(B) = \sum_i P(B \mid A_i)P(A_i),$$
where $\{A_i\}$ is a partition of the sample space.

Counting Techniques

- **Factorial:** $n! = n \times (n-1) \times \cdots \times 1$.
- **Permutations:** $P(n,k) = \frac{n!}{(n-k)!}$.
- **Combinations:** $C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Random Variables and Distributions

- **PMF:** $P(X = x), \quad \sum P(X = x) = 1$.
- **CDF:** $F(x) = P(X \leq x)$.

Expectation and Properties

- $E[X] = \sum xP(X = x)$.
- Linearity: $E[aX + b] = aE[X] + b$.
- $E[X + Y] = E[X] + E[Y]$.

Common Distributions

Bernoulli and Binomial Distributions

- **Bernoulli:** $P(X = x) = p^x(1-p)^{1-x}$, $x \in \{0, 1\}$
  - Expected Value: $E[X] = p$
  - Variance: $\text{Var}(X) = p(1-p)$
  - Examples:
    - Coin toss (Heads or Tails).
    - Success or failure in one trial.
    - Customer purchase (Yes/No).
- **Binomial:** $P(X = k) = \binom{n}{k}p^k(1-p)^{n-k}$, $k = 0, 1, \ldots, n$
  - Expected Value: $E[X] = np$
  - Variance: $\text{Var}(X) = np(1-p)$
  - Examples:
    - Number of successes in n trials.
    - Defective items in a batch.
    - Customers buying a product in n trials.

Hypergeometric Distribution

- **Hypergeometric:** $P(X = k) = \frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$
  - Expected Value: $E[X] = n\frac{M}{N}$
  - Variance: $\text{Var}(X) = n\frac{M}{N}\frac{N-M}{N}\frac{N-n}{N-1}$
  - Examples:
    - Sampling n balls from an urn without replacement.
    - Defective items in a random sample from a lot.
    - Hearts drawn in a card game.

Geometric and Negative Binomial Distributions

- **Geometric:** $P(X = k) = (1 - p)^{k-1}p$, $k \geq 1$
  - Expected Value: $E[X] = \frac{1}{p}$
  - Variance: $\text{Var}(X) = \frac{1-p}{p^2}$
  - Examples:
    * Number of trials until the first success.
    * Attempts needed to fix a bug.
    * Successful phone connection on trial k.
- **Negative Binomial:** $P(X = k) = \binom{k+r-1}{r-1}p^r(1 - p)^k$, $k \geq 0$
  - Expected Value: $E[X] = \frac{r(1-p)}{p}$
  - Variance: $\text{Var}(X) = \frac{r(1-p)}{p^2}$
  - Examples:
    * Failures before r successes in quality control.
    * Modeling r-th event (e.g., r-th success).
    * Incorrect guesses before r correct answers.

Uniform (Discrete) Distribution

- **Uniform (Discrete):** $P(X = x) = \frac{1}{n}$, $x \in \{x_1, \ldots, x_n\}$
  - Expected Value: $E[X] = \frac{x_1+x_n}{2}$
  - Variance: $\text{Var}(X) = \frac{(x_n-x_1+1)^2-1}{12}$
  - Examples:
    * Rolling a fair die (1–6).
    * Randomly selecting a card from a shuffled deck.
    * Equally likely lottery outcomes.