STAT 131 — Discussion Solutions (Lec 1 & Lec 2)

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How to read these solutions

Each item is solved in three short moves: **set up** the notation, show the **key idea** step-by-step, and **box** the final value or set.

Discussion (Lec 1)

1. Two balanced dice: sample space and "same value" event

Sample space. Let the first die be D_1 and the second D_2. The sample space is

$$\Omega = \{(i, j) : i \in \{1, 2, 3, 4, 5, 6\},\ j \in \{1, 2, 3, 4, 5, 6\}\}.$$

This has $|\Omega| = 6 \cdot 6 = 36$ outcomes.

Event that both rolls are the same.

$$E = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\} = \{(k, k) : k \in \{1, \dots, 6\}\}.$$

(Optional) If needed later, $|E| = 6$ and $P(E) = 6/36 = 1/6$ for fair dice.

2. Campus survey with two attributes

Let U = "undergraduate", G = "graduate" ($G = U^c$), O = "living off campus" and O^c = "living on campus". Universe size is $N = 60$. Given:

$$|U| = 36, \quad |O| = 9, \quad |U \cap O| = 3.$$

(a) Number who are undergraduates, or living off campus, or both. This is $|U \cup O|$.

$$|U \cup O| = |U| + |O| - |U \cap O| = 36 + 9 - 3 = \boxed{42}.$$

(b) Number of undergraduates living *on* campus. This is $|U \cap O^c| = |U| - |U \cap O| = 36 - 3 = \boxed{33}$.

(c) Number of graduate students living on campus. First $|G| = N - |U| = 60 - 36 = 24$. Also $|G \cap O| = |O| - |U \cap O| = 9 - 3 = 6$. Hence

$$|G \cap O^c| = |G| - |G \cap O| = 24 - 6 = \boxed{18}.$$

(Equivalently, all on-campus: $60 - 9 = 51$, subtract undergrad on-campus 33 to get 18.)

3. Set identities

(a) Show that $A = (A \cap B) \cup (A \cap B^c)$. *Proof.* For any ω, if $\omega \in A$, then either $\omega \in B$ or $\omega \in B^c$. In the first case $\omega \in A \cap B$; in the second, $\omega \in A \cap B^c$. Thus $A \subseteq (A \cap B) \cup (A \cap B^c)$. The reverse inclusion is immediate since both $A \cap B$ and $A \cap B^c$ are subsets of A. Hence equality. □

(b) About a likely typo and the intended identity. The statement often intended is: *If $B \subseteq A$, then* $A = B \cup (A \cap B^c)$. This is exactly part (a), and when $B \subseteq A$ we also have $A \cap B = B$, giving the useful partition $A = B \cup (A \setminus B)$ with disjoint pieces.

Note on the printed version. If one writes $A = B \cup (A \cap B)$ under the assumption $B \subseteq A$, then the right-hand side simplifies to B (since $A \cap B = B$), which would force $A = B$. So the version with B^c is the correct general identity.

Discussion (Lec 2)

1. Two balanced dice: probability the sum is odd

Let E be the event "sum is odd". A sum is odd iff one die shows an odd number and the other an even number. Each die has 3 odd and 3 even faces, and the dice are independent:

$$P(E) = P(\text{odd, even}) + P(\text{even, odd}) = \left(\frac{3}{6} \cdot \frac{3}{6}\right) + \left(\frac{3}{6} \cdot \frac{3}{6}\right) = \frac{1}{2}.$$

(Counting check: there are 18 odd-sum outcomes out of 36 total.)

2. At least one of two students fails

Let $A =$ "student 1 fails", $B =$ "student 2 fails". Given

$$P(A) = 0.5, \quad P(B) = 0.2, \quad P(A \cap B) = 0.1.$$

We want $P(A \cup B)$ (at least one failure). By inclusion–exclusion,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.2 - 0.1 = \boxed{0.6}.$$

(Complement check: $P(\text{neither fails}) = 1 - 0.6 = 0.4$.)

If a step feels fast, mark that specific line and ask about it in office hours.